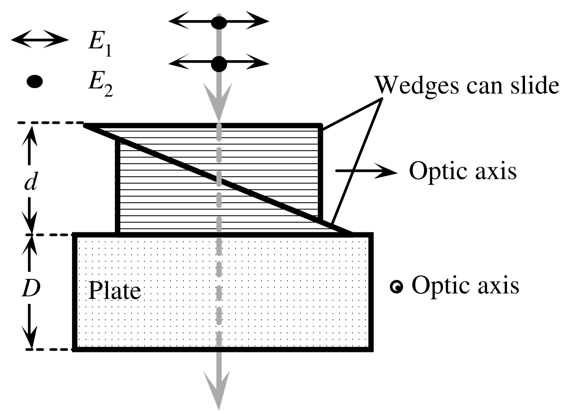
7.9 Soleil Compensator: Consider a Soleil compensator as in Figure 7Q9 that uses a quartz crystal. Given a light wave with a wavelength $\lambda \approx 600$ nm, a lower plate thickness of 5 mm, calculate the range of d values in Figure 7Q9 that provide a retardation from 0 to π (half-wavelength).



Soleil-Babinet Compensator

Figure 7Q9

Solution

The phase difference ϕ between the two polarizations passing through the Soleil-Babinet compensator

$$\phi = \frac{2\pi}{\lambda}(n_e - n_o)(D - d)$$

$\therefore$
$$d = D - \frac{\lambda\phi}{2\pi(n_e - n_o)}$$

When ϕ changes by $\Delta\phi$, then the change in d is Δd,

$$|\Delta d| = \left|\frac{\lambda\Delta\phi}{2\pi(n_e - n_o)}\right|$$

$\Delta\phi = \pi$, $|\Delta d| = \left|\frac{\lambda\Delta\phi}{2\pi(n_e - n_o)}\right| = \frac{(600\times10^{-6}\ \text{mm})(\pi)}{2\pi(1.5533 - 1.5442)} = 0.033$ mm =**33 microns**

7.15 Transverse Pockels cell with $LiNbO_3$ Suppose that instead of the configuration in Figure 7.20, the field is applied along the z-axis of the crystal, the light propagates along the y-axis. The z-axis is the polarization of the ordinary wave and x-axis that of the extraordinary wave. Light propagates through as o- and e-waves. Given that $E_a = V/d$, where d the crystal length along z, the indices are

$$n_o' \approx n_o + \tfrac{1}{2}n_o^3 r_{13}E_a \qquad \text{and} \qquad n_e' \approx n_e + \tfrac{1}{2}n_e^3 r_{33}E_a$$

Show that the phase difference between the *o*- and *e*-waves emerging from the crystal is,

$$\Delta\phi = \phi_e - \phi_o = \frac{2\pi L}{\lambda}\left(n_e - n_o\right) + \frac{2\pi L}{\lambda}\frac{1}{2}\left(n_e^3 r_{33} - n_o^3 r_{13}\right)\frac{V}{d}$$

where L is the crystal length along the y-axis.

Explain the first and second terms. How would you use two such Pockels cells to cancel the first terms in the total phase shift for the two cells.

If the light beam entering the crystal is linearly polarized in the z-direction, show that

$$\Delta\phi = \frac{2\pi n_e L}{\lambda} + \frac{2\pi L}{\lambda}\frac{(n_e^3 r_{33})}{2}\frac{V}{d}$$

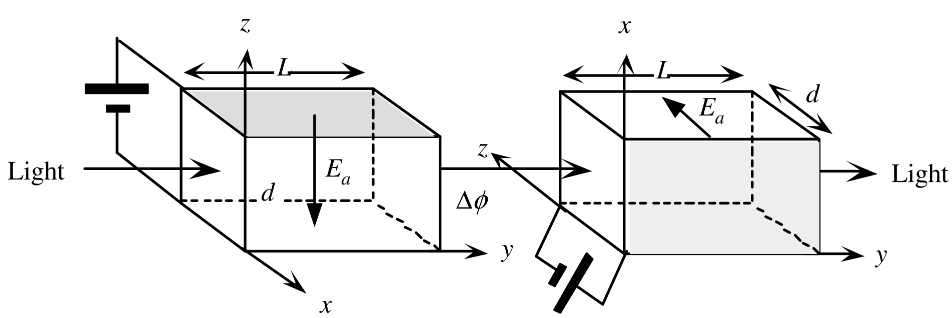
Consider a nearly monochromatic light beam of the free-space wavelength $\lambda = 500$ nm and polarization along z-axis. Calculate the voltage V_π needed to *change* the output phase $\Delta\phi$ by π given a $LiNbO_3$ crystal with $d/L = 0.01$ (see table 7.2).

Solution

Consider the phase change between the two electric field components,

$$\Delta\phi = \phi_e - \phi_o = \frac{2\pi L}{\lambda}\left(n_e - n_o\right) + \frac{2\pi L}{\lambda}\frac{1}{2}\left(n_e^3 r_{33} - n_o^3 r_{13}\right)\frac{V}{d}$$

The first term is the *natural* birefringence of the crystal, just as in the calcite crystal, and occurs all the time, even without an applied field. The second term is the Pockels effect, applied field inducing a change in the refractive indices. Figure 7Q15 shows how two Pockels cells may be used to cancel the first terms in the combined system.



Two tranverse Pockels cell phase modulators together cancel the natural birefringence in each crystal.

Figure 7Q15

If the light beam is linearly polarized with its field along z, we only need to consider the extraordinary ray, thus we can set $\phi_o = 0$.

$$\Delta\phi = \phi_e = \frac{2\pi L n_e}{\lambda} + \frac{2\pi L}{\lambda}\frac{1}{2}\left(n_e^3 r_{33}\right)\frac{V}{d}$$

The first term does not depend on the voltage. The voltage V that **changes** the output phase by π is

$$\frac{2\pi L}{\lambda}\frac{1}{2}\left(n_e^3 r_{33}\right)\frac{V}{d} = \pi$$

or $$V = \frac{d}{L}\left(\frac{\lambda}{n_e^3 r_{33}}\right) = (0.01)\left(\frac{500\times10^{-9}\ \text{m}}{(2.187)^3(30.8\times10^{-12}\ \text{m/V})}\right)$$

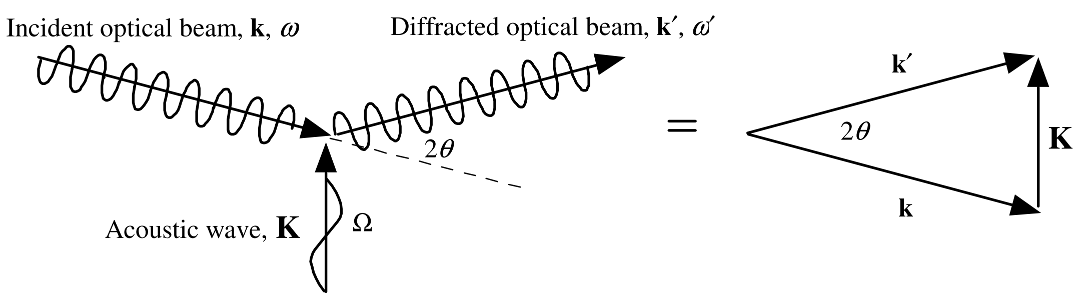
∴ V = **15.5 V**

7.18 Bragg acousto-optic modulator diffraction: Consider an acousto-optic modulator. We can represent the incident and diffracted optical waves in terms of their wavevectors **k** and **k**′. The incident and the diffracted photons will have energies $\hbar\omega$ and $\hbar\omega'$ and momenta $\hbar k$ and $\hbar k'$. An acoustic wave is consists of lattice vibrations (vibrations of the crystal atoms) and these vibrations are quantized just like electromagnetic waves are quantized in terms of photons. A quantum of lattice vibration is called a **phonon**. A traveling lattice wave is essentially a strain wave and can be represented by $S = S_o\cos(\Omega t - Kx)$ where S is the instantaneous strain at x, Ω is the angular acoustic frequency, and K is the wavevector, $K = 2\pi/\Lambda$ and S_o is the amplitude of the strain wave. A phonon has an energy $\hbar\Omega$ and a momentum $\hbar K$. When an incoming photon is diffracted, it does so by interacting with a phonon; it can absorb or generate a phonon. We can treat the interactions as we do between any two particles; they must obey the conservation of momentum and energy rules:

$$\hbar\mathbf{k}' = \hbar\mathbf{k} \pm \hbar\mathbf{K}$$

$$\hbar\omega' = \hbar\omega \pm \hbar\Omega$$

The positive sign case is illustrated in Figure 7Q18 which involves absorbing a phonon. Since the acoustic frequencies are orders of magnitude smaller than optical frequencies ($\Omega << \omega$), we can assume that in magnitude $k' \approx k$. Hence using the above rules, which correspond to Figure 7Q18 derive the Bragg diffraction condition.



Wavevectors for the incident and diffracted optical waves and the acoustic wave.

Figure 7Q18

Solution

Using $k' \approx k$ in Figure 7Q18 we get,

$$K = 2k\sin\theta$$

We can substitute for the photon and phonon wavevectors in terms of phonon wavelength Λ and free space photon wavelength λ,

$$\frac{2\pi}{\Lambda} = 2\left(\frac{2\pi n}{\lambda}\right)\sin\theta$$

which leads to the Bragg condition

$$2\Lambda\sin\theta = \lambda/n.$$

7.20 **SHG** The mismatch between k_2 for the second harmonic wavevector and k_1 for the fundamental wave is defined by $\Delta k = k_2 - 2k_1$. Perfect match means $k_2 = 2k_1$ and $\Delta k = 0$. When $\Delta k \neq 0$, then the coherence length l_c is given by $l_c = \pi/(\Delta k)$. Show that

$$l_c = \frac{\lambda}{4(n_2 - n_1)}$$

where λ is the free-space wavelength of the fundamental wave. Suppose that a light with wavelength 1000 nm is passed through KDP crystal along its optic axis. Given that $n_o = 1.509$ at $\lambda = 1000$ nm and $n_o = 1.530$ nm at 2λ, what is the coherence length l_c? Find the percentage difference between n_2 and n_1 for a coherence length of 2 mm?

Solution

Let ω be the frequency of the fundamental and 2ω that of the second harmonic. Free space wavelength $\lambda = c/\upsilon = [(2\pi c)/\omega]$ where υ is the frequency and $\omega = 2\pi\upsilon$. Then,

$$\Delta k = k_2 - 2k_1 = \frac{(2\omega)n_2}{c} - 2\left[\frac{(\omega)n_1}{c}\right] = 2\frac{\omega}{c}(n_2 - n_1)$$

and substituting in the expression $l_c = \pi/\Delta k$, we get,

$$l_c = \frac{\pi}{\Delta k} = \frac{\pi}{2\frac{\omega}{c}(n_2 - n_1)} = \frac{\pi}{2\frac{2\pi}{\lambda}(n_2 - n_1)} = \frac{\lambda}{4(n_2 - n_1)}$$

Substituting $\lambda = 1000$ nm = 1 µm, $n_1 = 1.509$, $n_2 = 1.53$, we find

$$l_c = \frac{\lambda}{4(n_2 - n_1)} = \frac{1\times10^{-6}}{4(1.53 - 1.509)} = \mathbf{11.9\ \mu m.}$$

Consider,

$$\frac{(n_2 - n_1)}{n_1} = \frac{\lambda}{4l_c n_1} = \frac{1\times10^{-6}}{4(2\times10^{-3})(1.509)} = 8.2\times10^{-5} \text{ or } \mathbf{8.2\times10^{-3}\ \%}$$